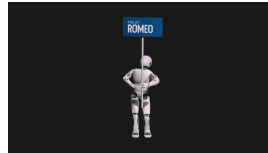
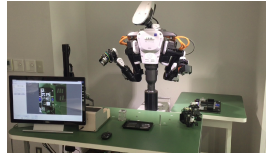
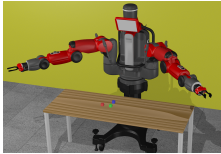


Manipulation motion planning

Florent Lamiraux

CNRS-LAAS, Toulouse, France

A few examples



Definitions

A manipulation motion

- ▶ is the motion of
 - ▶ one or several robots and of
 - ▶ one or several objects
- ▶ such that each object
 - ▶ either is in a stable position, or
 - ▶ is moved by one or several robots.

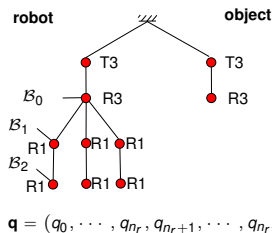
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Composite robot

Kinematic chain composed of each robot and of each object

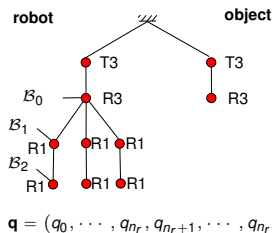


The configuration space of a composite robot is the cartesian product of the configuration spaces of each robot and object.

$$\mathcal{C} = \mathcal{C}_{r1} \times \mathcal{C}_{r_{nb \text{ robots}}} \times SE(3)^{nb \text{ objects}}$$

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Numerical constraints

Constraints to which manipulation motions are subject can be expressed numerically.

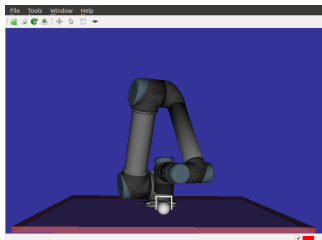
- ▶ Numerical constraints:

$$f(\mathbf{q}) = 0, \quad \begin{array}{l} m \in \mathbb{N}, \\ f \in \mathcal{C}^1(\mathcal{C}, \mathbb{R}^m) \end{array}$$

- ▶ Parameterizable numerical constraints:

$$f(\mathbf{q}) = f_0, \quad \begin{array}{l} m \in \mathbb{N}, \\ f \in \mathcal{C}^1(\mathcal{C}, \mathbb{R}^m) \\ f_0 \in \mathbb{R}^m \end{array}$$

Example: robot manipulating a ball



$$\mathcal{C} = [-\pi, \pi]^6 \times \mathbb{R}^3 \quad (1)$$

$$\mathbf{q} = (q_0, \dots, q_5, x_b, y_b, z_b) \quad (2)$$

Two *states*:

- ▶ **placement**: the ball is lying on the table,
- ▶ **grasp**: the ball is hold by the end-effector.

Example: robot manipulating a ball

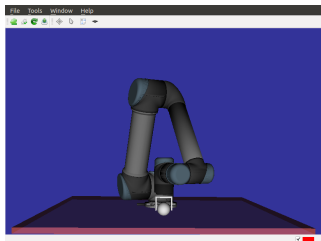
Each state is defined by a numerical constraint

► placement

$$z_b = 0$$

► grasp

$$\mathbf{x}_{gripper}(q_0, \dots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$



Each state is a sub-manifold of the configuration space

Example: robot manipulating a ball

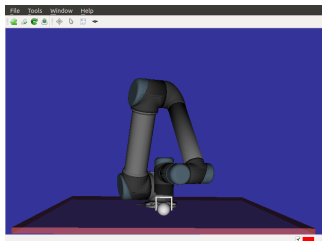
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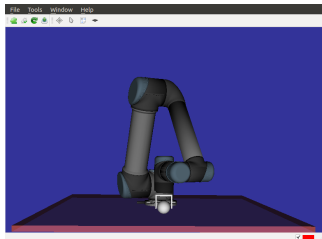
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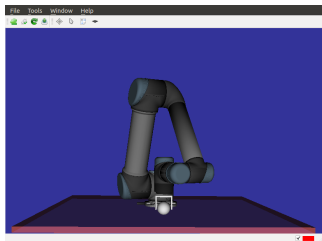


Two *types of motion*:

- ▶ *transit*: the ball is lying and **fixed** on the table,
- ▶ *transfer*: the ball moves with the end-effector.

Example: robot manipulating a ball

Motion constraints



► transit

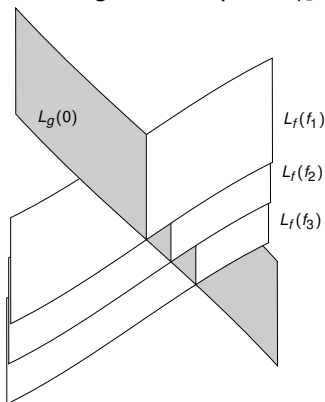
$$\begin{array}{lcl} x_b = x_0 & & \\ y_b = y_0 & \} & \text{parameterizable} \\ z_b = 0 & \} & \text{simple} \end{array}$$

► transfer

$$\mathbf{x}_{gripper}(q_0, \dots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Foliation

Motion constraints define a foliation of the admissible configuration space ($\text{grasp} \cup \text{placement}$).



- f : position of the ball

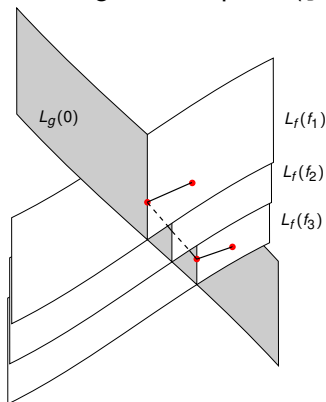
$$L_f(f_1) = \{\mathbf{q} \in \mathcal{C}, f(\mathbf{q}) = f_1\}$$

- g : grasp of the ball

$$L_g(0) = \{\mathbf{q} \in \mathcal{C}, g(\mathbf{q}) = 0\}$$

Foliation

Motion constraints define a foliation of the admissible configuration space ($\text{grasp} \cup \text{placement}$).



Solution to a manipulation planning problem is a concatenation of *transit* and *transfer* paths.

General case

In a manipulation problem,

- ▶ the state of the system is subject to
 - ▶ numerical constraints
- ▶ trajectories of the system are subject to
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 - ▶ parameterizable numerical constraints.

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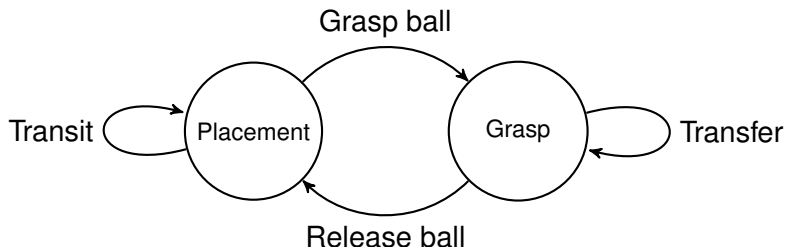
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 - ▶ parameter value is constant along trajectories.

Constraint graph

A manipulation planning problem can be represented by a *manipulation graph*.

- ▶ **Nodes** or *states* are numerical constraints.
- ▶ **Edges** or *transitions* are parameterizable numerical constraints.



Projecting configuration on constraint

Newton-Raphson algorithm

- ▶ \mathbf{q}_0 configuration,
- ▶ $f(\mathbf{q}) = 0$ non-linear constraint,
- ▶ ϵ numerical tolerance

Projection (\mathbf{q}_0, f):

$\mathbf{q} = \mathbf{q}_0; \alpha = 0.95$

for i from 1 to max_iter:

$$\mathbf{q} = \mathbf{q} - \alpha \left(\frac{\partial f}{\partial \mathbf{q}}(\mathbf{q}) \right)^+ f(\mathbf{q})$$

if $\|f(\mathbf{q})\| < \epsilon$: return \mathbf{q}

return failure

Steering method

Mapping \mathcal{SM} from $\mathcal{C} \times \mathcal{C}$ to $C^1([0, 1], \mathcal{C})$ such that

$$\mathcal{SM}(\mathbf{q}_0, \mathbf{q}_1)(0) = \mathbf{q}_0$$

$$\mathcal{SM}(\mathbf{q}_0, \mathbf{q}_1)(1) = \mathbf{q}_1$$

Constrained steering method

Let

- ▶ \mathcal{SM} be a steering method
- ▶ $f \in C^1(\mathcal{C}, \mathbb{R}^m)$ be a numerical constraint.

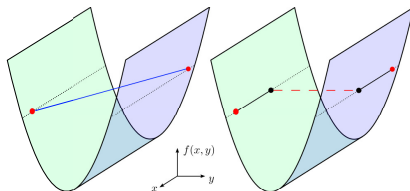
A constrained steering method $\bar{\mathcal{SM}}$ of constraint f is a steering method such that

$$\forall t \in [0, 1], f(\bar{\mathcal{SM}}(t)) = 0$$

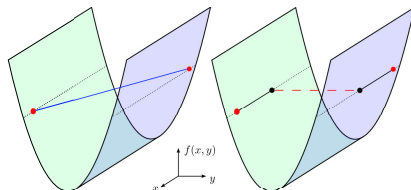
Projecting path on constraint

- ▶ *path*: mapping from $[0, 1]$ to \mathcal{C}
- ▶ $f(\mathbf{q}) = 0$ non-linear constraint,

Applying Newton Raphson at each point may result in a discontinuous path



Discontinuous Projection



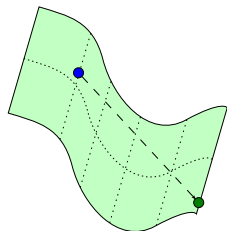
$$\mathcal{C} = \mathbb{R}^2, f(x, y) = y^2 - 1$$

$$\frac{\partial f}{\partial \mathbf{q}} = \begin{pmatrix} 0 & 2y \end{pmatrix}, \quad \frac{\partial f^+}{\partial \mathbf{q}} = \begin{pmatrix} 0 \\ \frac{1}{2y} \end{pmatrix} \text{ ou } \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$y_{i+1} = y_i + \frac{1 - y_i^2}{2y_i}$$

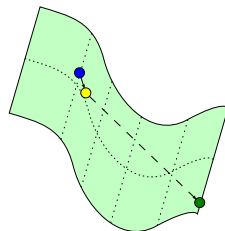
Testing projection continuity

- ▶ The initial path is sampled and successive samples are projected,



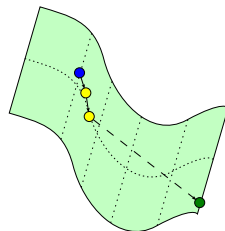
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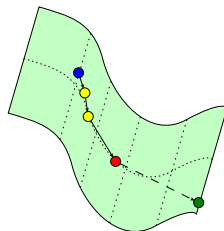
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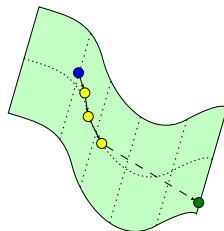
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- ▶ The initial path is sampled and successive samples are projected,
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- ▶



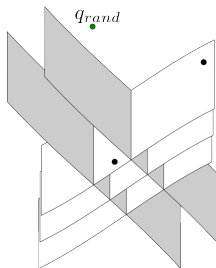
Testing projection continuity

- ▶ The initial path is sampled and successive samples are projected,
- ▶ if 2 successive projections are too far away, an intermediate sample is selected.
- ▶ Choosing appropriate sampling ensures us continuity of the projection.



Algorithm

Manipulation RRT



Manipulation RRT

$\mathbf{q}_{rand} = \text{shoot_random_config}()$

for each connected component:

$\mathbf{q}_{near} = \text{nearest_neighbour}(\mathbf{q}_{rand}, \text{roadmap})$

$e = \text{select_transition}(\mathbf{q}_{near})$

$\mathbf{q}_{proj} = \text{generate_target_config}(\mathbf{q}_{near}, \mathbf{q}_{rand}, e)$

$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, \text{edge})$

$\text{roadmap.insert_node}(\mathbf{q}_{new})$

$\text{roadmap.insert_edge}(e, \mathbf{q}_{near}, \mathbf{q}_{new})$

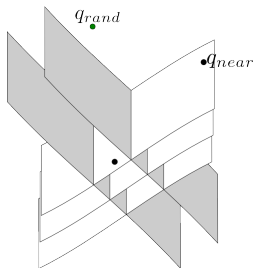
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for $\mathbf{q} \in (\mathbf{q}_{new}^1, \dots, \mathbf{q}_{new}^{n_{cc}})$:

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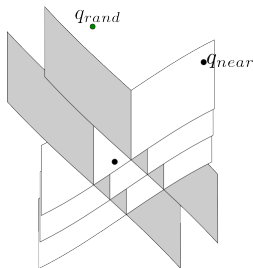
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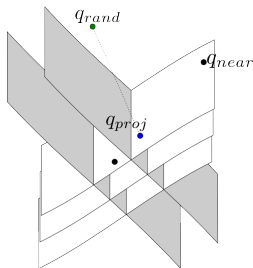
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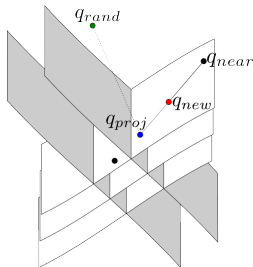
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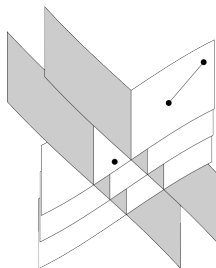
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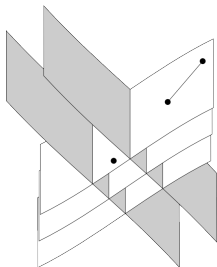
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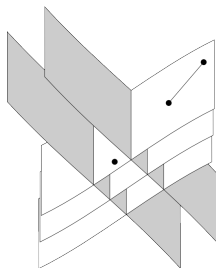
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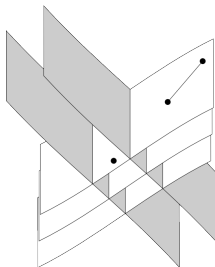
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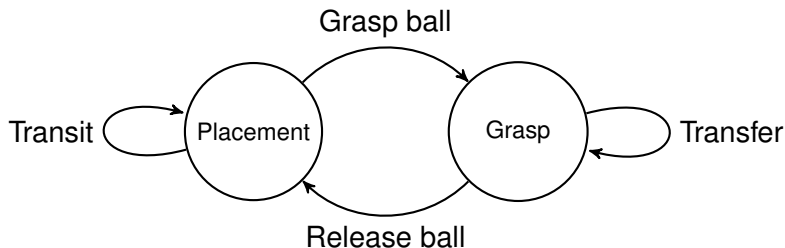
for $\mathbf{q} \in (\mathbf{q}_{new}^1, \dots, \mathbf{q}_{new}^{n_{cc}})$:

connect (\mathbf{q} , roadmap)

Select transition

$e = \text{select_transition}(\mathbf{q}_{near})$

Outward edges of each node are given a probability distribution. The transition from a node to another node is chosen by random sampling.



Generate target configuration

$$\mathbf{q}_{proj} = \text{generate_target_config}(\mathbf{q}_{near}, \mathbf{q}_{rand}, e)$$

Once edge e has been selected, \mathbf{q}_{rand} is *projected* onto the destination node n_{dest} in a configuration reachable by \mathbf{q}_{near} .

$$\begin{aligned}f_e(\mathbf{q}_{proj}) &= f_e(\mathbf{q}_{near}) \\f_{dest}(\mathbf{q}_{proj}) &= 0\end{aligned}$$

Extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, \text{edge})$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on edge constraint:

- ▶ if projection successful and projected path collision free

$$\mathbf{q}_{new} \leftarrow \mathbf{q}_{proj}$$

- ▶ otherwise $(\mathbf{q}_{near}, \mathbf{q}_{new}) \leftarrow$ largest path interval tested as collision-free with successful projection.

$$\forall \mathbf{q} \in (\mathbf{q}_{near}, \mathbf{q}_{new}), f_e(\mathbf{q}) = f_e(\mathbf{q}_{near})$$

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Connect

connect (\mathbf{q} , roadmap)

for each connected component cc not containing \mathbf{q} :

for all n closest config \mathbf{q}_1 to \mathbf{q} in cc :

▶ connect (\mathbf{q} , \mathbf{q}_1)

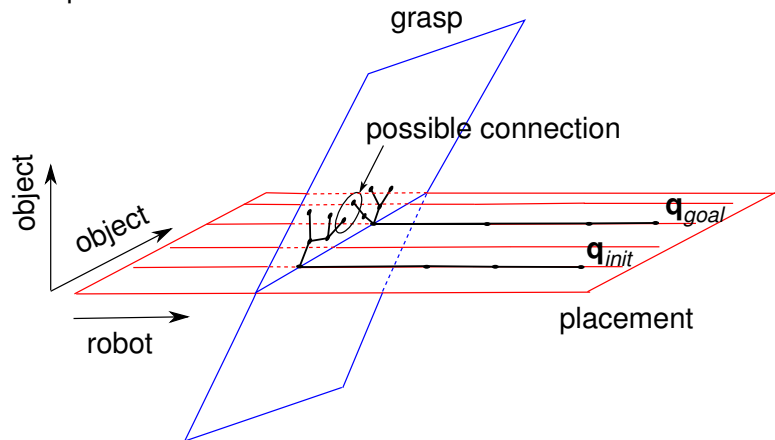
Connect

```
connect ( $\mathbf{q}_0$ ,  $\mathbf{q}_1$ ):  
     $s_0$  = state ( $\mathbf{q}_0$ )  
     $s_1$  = state ( $\mathbf{q}_1$ )  
     $e$  = transition ( $n_0$ ,  $n_1$ )  
    if  $e$  and  $f_e(\mathbf{q}_0) == f_e(\mathbf{q}_1)$ :  
        if  $p$  = projected_path ( $e$ ,  $\mathbf{q}_0$ ,  $\mathbf{q}_1$ ) collision-free:  
            roadmap.insert_edge ( $e$ ,  $\mathbf{q}_0, \mathbf{q}_1$ )  
  
    return
```

Connecting trees

Manipulation RRT is initialized with \mathbf{q}_{init} , \mathbf{q}_{goal} .

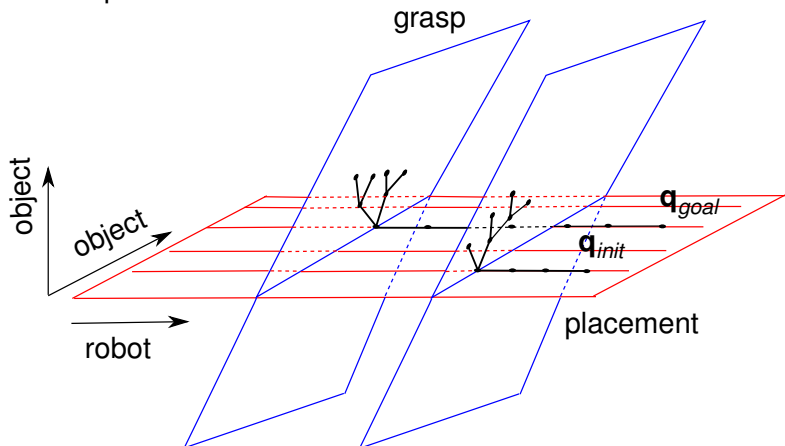
- ▶ 2 connected components.
- ▶ possible connection.



Connecting trees: general case

Manipulation RRT is initialized with \mathbf{q}_{init} , \mathbf{q}_{goal} .

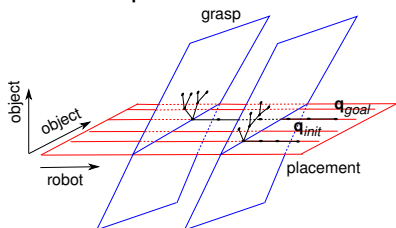
- ▶ 2 connected components,
- ▶ no possible connection.



Connecting trees: general case

Manipulation RRT is initialized with \mathbf{q}_{init} , \mathbf{q}_{goal} .

- ▶ 2 connected components,
- ▶ no possible connection.



Crossed foliation transition: generate target configuration

$\mathbf{q}_{proj} =$
`generate_target_config`(\mathbf{q}_{near} , \mathbf{q}_{rand} , e)

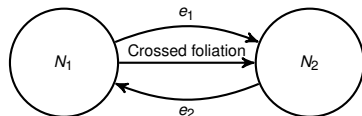
$\mathbf{q}_1 \leftarrow$ pick configuration

- ▶ in node N_1 ,
- ▶ not in same connected component as \mathbf{q}_{near}

$$f_{e_1}(\mathbf{q}_{proj}) = f_{e_1}(\mathbf{q}_{near})$$

$$f_{e_2}(\mathbf{q}_{proj}) = f_{e_2}(\mathbf{q}_1)$$

$$f_{N_2}(\mathbf{q}_{proj}) = 0$$



Crossed foliation transition: extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, e_1)$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on e_1 constraint:

- ▶ if projection successful and projected path collision free

$$\mathbf{q}_2 \leftarrow \mathbf{q}_{proj}$$

$$f_{e_2}(\mathbf{q}_2) = f_{e_2}(\mathbf{q}_1)$$

$$f_{N_2}(\mathbf{q}_2) = 0$$

- ▶ \mathbf{q}_2 is connectable to \mathbf{q}_1 via e_2 .

Crossed foliation transition: extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, e_1)$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on e_1 constraint:

- ▶ if projection successful and projected path collision free

$$\mathbf{q}_2 \leftarrow \mathbf{q}_{proj}$$

$$f_{e_2}(\mathbf{q}_2) = f_{e_2}(\mathbf{q}_1)$$

$$f_{N_2}(\mathbf{q}_2) = 0$$

- ▶ \mathbf{q}_2 is connectable to \mathbf{q}_1 via e_2 .

Relative positions as numerical constraints

Let $T_1 = T_{(R_1, t_1)}$ and $T_2 = T_{(R_2, t_2)}$ be two rigid-body transformations. The relative transformation $T_{2/1} = T_1^{-1} \circ T_2$ can be represented by a vector of dimension 6:

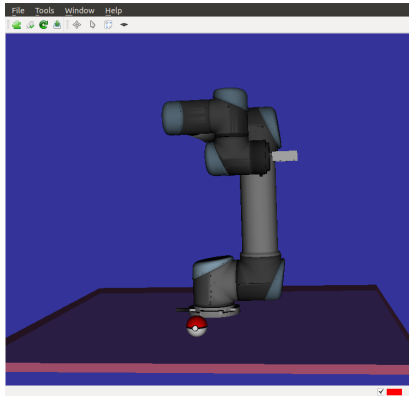
$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

where

$$\mathbf{u} = R_1^T(t_2 - t_1)$$

$R_1^T R_2$ is the matrix of the rotation around axis $\mathbf{v}/\|\mathbf{v}\|$ and of angles $\|\mathbf{v}\|$.

A few words about the BE



Thèse CIFRE sur les robots parallèles à câbles

En collaboration avec Tecnalía France



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