

Motion planning

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Task and Motion Planning

Definition

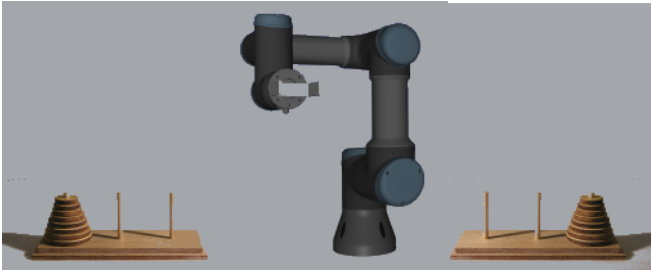


Given

- ▶ One or several robots,
- ▶ One or several objects,
- ▶ initial configurations for robots and objects
- ▶ goal configurations for robots and objects

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Configuration space

Configuration

$$\mathbf{q} = (\mathbf{q}_{r_1}, \mathbf{q}_{r_2}, \mathbf{q}_{c_1}, \mathbf{q}_{c_2}, \mathbf{q}_{s_1}, \mathbf{q}_{s_2})$$

$$\mathbf{q}_{r_1}, \mathbf{q}_{r_2} \in \mathbb{R}^6$$

$$\mathbf{q}_{c_1}, \mathbf{q}_{c_2}, \mathbf{q}_{s_1}, \mathbf{q}_{s_2} \in \mathbb{R}^7$$

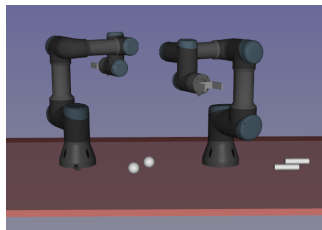
where

$\mathbf{q}_{r_1} = (q_1, \dots, q_6)$ is the vector of joint angles,

$\mathbf{q}_{c_1} = (x, y, z, X, Y, Z, W)$

$W + Xi + Yj + Zk$ is a **unit** quaternion.

The configuration space is a differential manifold.



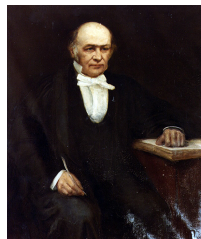
Quaternions

Non-commutative field isomorphic to $\mathbb{R}^4$, spanned by three elements i, j, k that satisfy the following relations :

$$i^2 = j^2 = k^2 = ijk = -1$$

from which we immediately deduce

$$ij = k, \quad jk = i, \quad ki = j$$



Hamilton (1843)

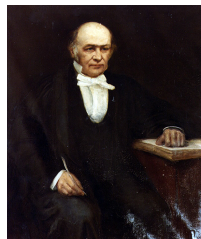
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Unit Quaternions and rotations

Let $q = q_0 + q_1i + q_2j + q_3k$ be a unit quaternion :

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$$

$\forall x = (x_0, x_1, x_2) \in \mathbb{R}^3$, let $u = x_0i + x_1j + x_2k$

$$q \cdot u \cdot q^* = y_0i + y_1j + y_2k$$

where $q^* = q_0 - q_1i - q_2j - q_3k$ is the conjugate of q .

$y = (y_0, y_1, y_2)$ is the image of x by the rotation of matrix

$$\begin{pmatrix} 1 - 2(q_2^2 + q_3^2) & 2q_2q_1 - 2q_3q_0 & 2q_3q_1 + 2q_2q_0 \\ 2q_2q_1 + 2q_3q_0 & 1 - 2(q_1^2 + q_3^2) & 2q_3q_2 - 2q_1q_0 \\ 2q_3q_1 - 2q_2q_0 & 2q_3q_2 + 2q_1q_0 & 1 - 2(q_1^2 + q_2^2) \end{pmatrix}$$

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Unit Quaternions and rotations

- ▶ Notice that q and $-q$ represent the same rotation

Definitions

- ▶ **Workspace** : $\mathcal{W} = \mathbb{R}^2$ or $\mathbb{R}^3$: space in which the robot evolves
- ▶ **Obstacle in workspace** : compact subset of $\mathcal{W}$, denoted by $\mathcal{O}$.
- ▶ **Configuration space** : $\mathcal{C}$.
- ▶ **Position in configuration** $\mathbf{q}$ of a point $M \in \mathcal{B}_i$: $\mathbf{x}_i(M, \mathbf{q})$.
- ▶ **Obstacle in the configuration space** :

$$\begin{aligned}\mathcal{C}_{obst} = \{ & \mathbf{q} \in \mathcal{C}, \quad \exists i \in \{1, \dots, m\}, \exists M \in \mathcal{B}_i, \mathbf{x}_i(M, \mathbf{q}) \in \mathcal{O} \text{ or} \\ & \exists i, j \in \{1, \dots, m\}, \exists M_i \in \mathcal{B}_i, \exists M_j \in \mathcal{B}_j, \\ & \mathbf{x}_i(M_i, \mathbf{q}) = \mathbf{x}_j(M_j, \mathbf{q}) \}\end{aligned}$$

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Motion

- ▶ Configuration space :
 - ▶ differential manifold
- ▶ Motion :
 - ▶ continuous function from $[0, 1]$ to $\mathcal{C}$.
- ▶ Collision-free motion :
 - ▶ continuous function from $[0, 1]$ to $\mathcal{C}_{free}$.

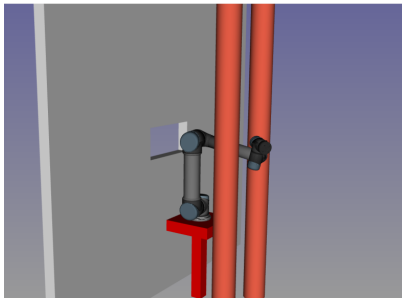
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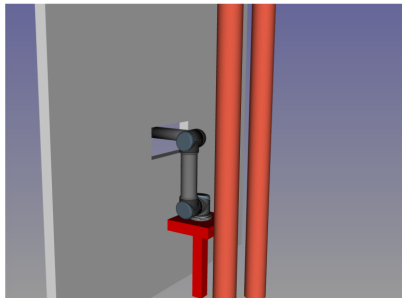
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Motion planning problem



initial configuration



goal configuration

$$\mathcal{C} = \mathbb{R}^6$$

History

- ▶ Before the 1990's : mainly a mathematical problem
 - ▶ real algebraic geometry,
 - ▶ decidability : Schwartz and Sharir 1982,
 - ▶ Tarski theorem, Collins decomposition,
- ▶ from the 1990's : an algorithmic problem
 - ▶ random sampling (1993),
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Random methods

- ▶ In the early 1990's, random methods started being developed
- ▶ Principle
 - ▶ shoot random configurations
 - ▶ test whether they are in collision
 - ▶ build a graph (roadmap) the nodes of which are free configurations
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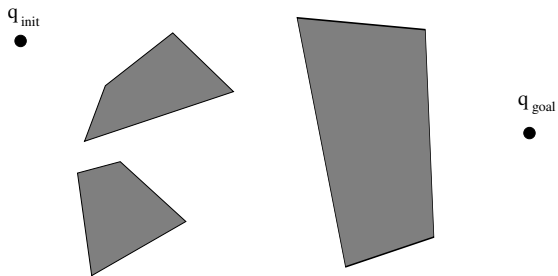
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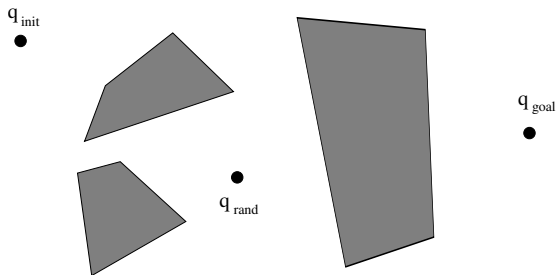
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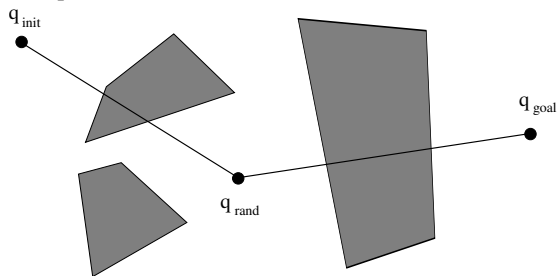
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Pick a random configuration



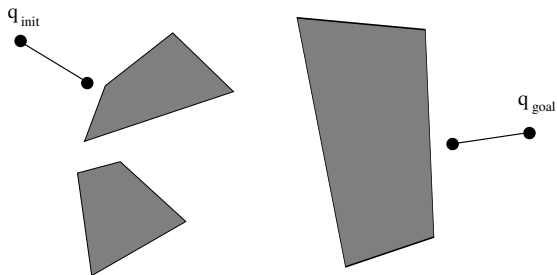
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Try to connect it to the nearest nodes of each connected component



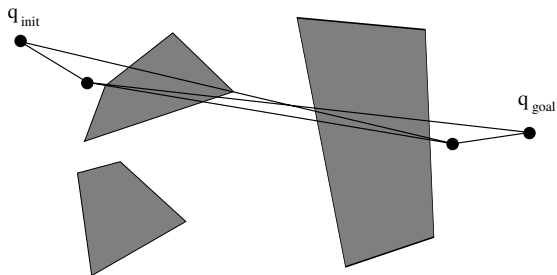
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Keep collision-free parts of paths

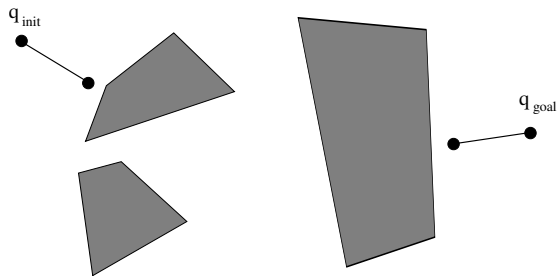


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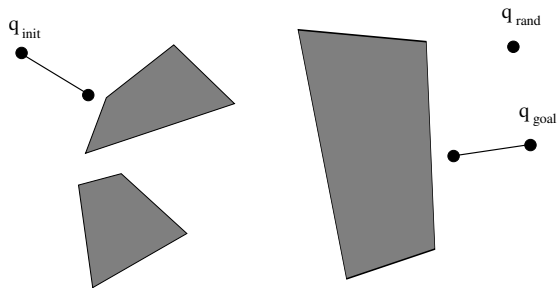
Try to connect new nodes to nearest nodes of other connected components



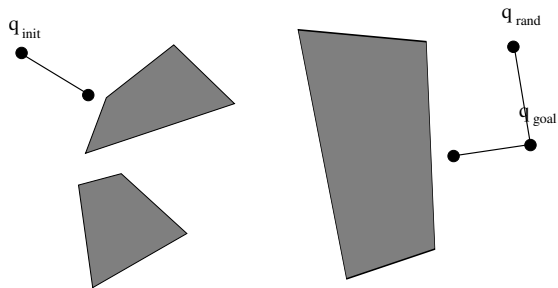
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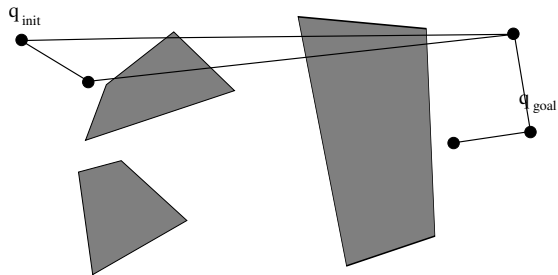
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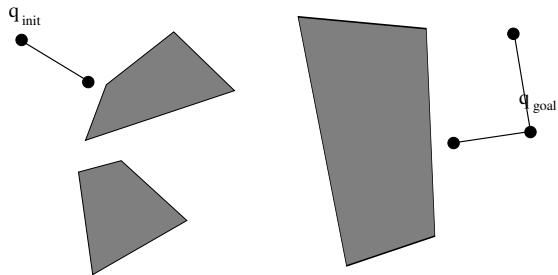
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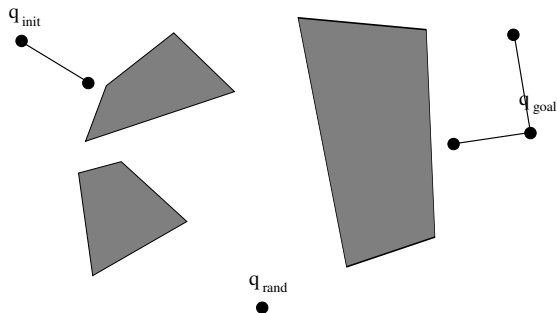
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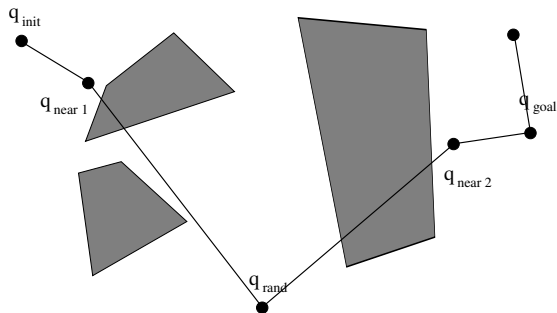
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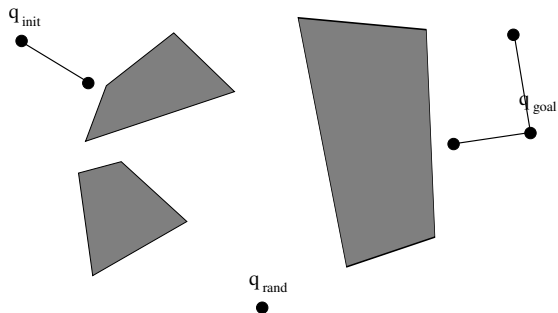
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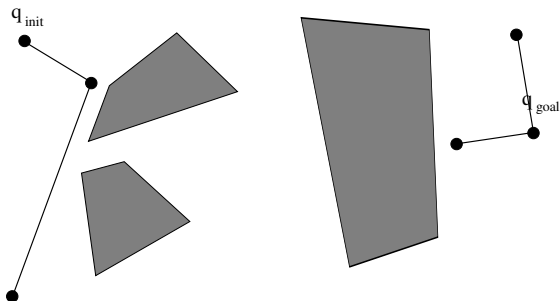
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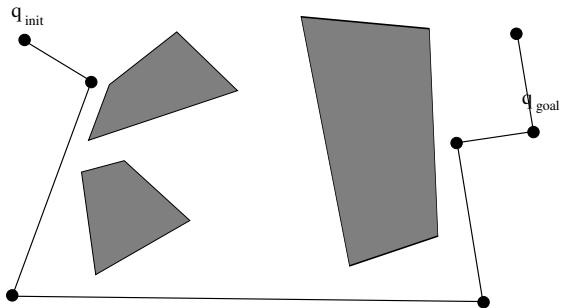
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Random methods

- ▶ Pros :
 - ▶ no explicit computation of the free configuration space,
 - ▶ easy to implement,
 - ▶ robust.
- ▶ Cons :
 - ▶ no completeness property, only probabilistic completeness,
 - ▶ difficult to find narrow passages.
- ▶ Requested operators :
 - ▶ Collision tests
 - ▶ for configurations (static),
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A manipulation motion

- ▶ is the motion of
 - ▶ one or several robots and of
 - ▶ one or several objects
- ▶ such that each object
 - ▶ either is in a stable position, or
 - ▶ is moved by one or several robots.

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Numerical constraints

Constraints to which manipulation motions are subject can be expressed numerically.

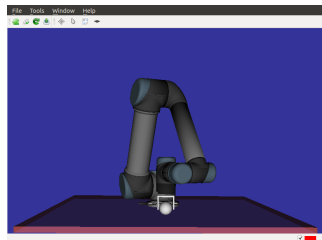
- ▶ Numerical constraints :

$$f(\mathbf{q}) = 0, \quad \begin{array}{l} m \in \mathbb{N}, \\ f \in C^1(\mathcal{C}, \mathbb{R}^m) \end{array}$$

- ▶ Parameterizable numerical constraints :

$$f(\mathbf{q}) = f_0, \quad \begin{array}{l} m \in \mathbb{N}, \\ f \in C^1(\mathcal{C}, \mathbb{R}^m) \\ f_0 \in \mathbb{R}^m \end{array}$$

Example : robot manipulating a ball



$$\mathcal{C} = [-\pi, \pi]^6 \times \mathbb{R}^3 \quad (1)$$

$$\mathbf{q} = (q_0, \dots, q_5, x_b, y_b, z_b) \quad (2)$$

Two *states* :

- ▶ placement : the ball is lying on the table,
- ▶ grasp : the ball is held by the end-effector.

Example : robot manipulating a ball

Each state is defined by a numerical constraint

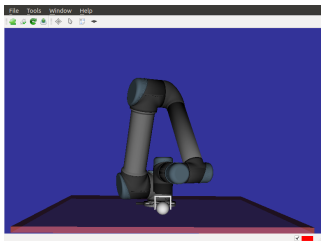
► placement

$$z_b = 0$$

► grasp

$$\mathbf{x}_{gripper}(q_0, \dots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Each state is a sub-manifold of the configuration space



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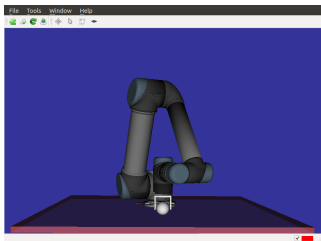
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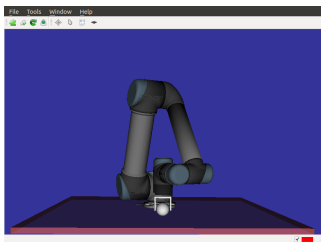
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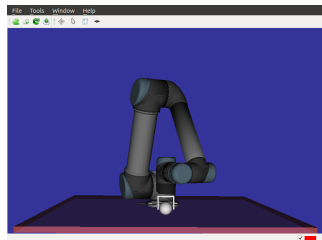


Two *types of motion* :

- ▶ transit : the ball is lying and **fixed** on the table,
- ▶ transfer : the ball moves with the end-effector.

Example : robot manipulating a ball

Motion constraints



► transit

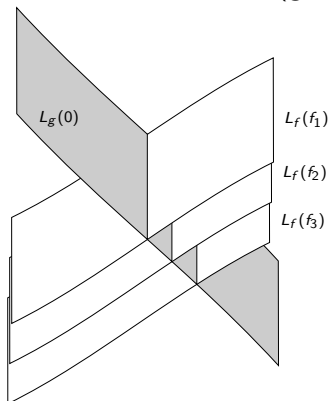
$$\begin{array}{rcl} x_b & = & x_0 \\ y_b & = & y_0 \\ z_b & = & 0 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{parameterizable} \\ \text{simple} \end{array}$$

► transfer

$$\mathbf{x}_{gripper}(q_0, \dots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Foliation

Motion constraints define a foliation of the admissible configuration space ($\text{grasp} \cup \text{placement}$).



- f : position of the ball

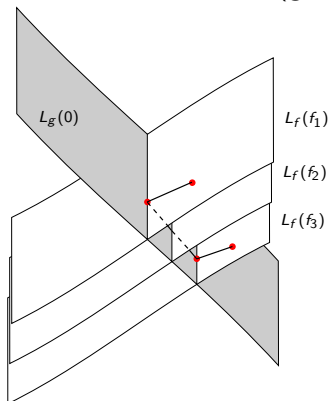
$$L_f(f_1) = \{\mathbf{q} \in \mathcal{C}, f(\mathbf{q}) = f_1\}$$

- g : grasp of the ball

$$L_g(0) = \{\mathbf{q} \in \mathcal{C}, g(\mathbf{q}) = 0\}$$

Foliation

Motion constraints define a foliation of the admissible configuration space ($\text{grasp} \cup \text{placement}$).



Solution to a manipulation planning problem is a concatenation of *transit* and *transfer* paths.

General case

In a manipulation problem,

- ▶ the state of the system is subject to
 - ▶ numerical constraints
- ▶ trajectories of the system are subject to
 - ▶ numerical constraints
 - ▶ parameterizable numerical constraints.

General case

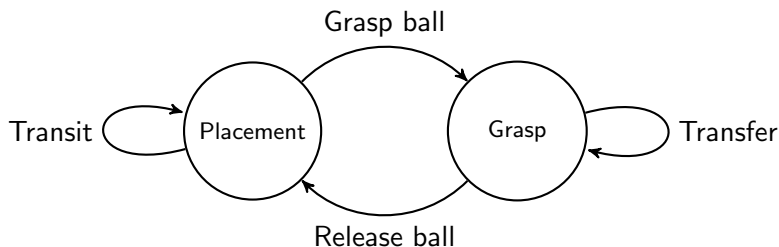
In a manipulation problem,

- ▶ the state of the system is subject to
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- ▶ trajectories of the system are subject to
 - ▶ numerical constraints
 - ▶ parameterizable numerical constraints.

Constraint graph

A manipulation planning problem can be represented by a *manipulation graph*.

- ▶ **Nodes** or *states* are numerical constraints.
- ▶ **Edges** or *transitions* are parameterizable numerical constraints.



Projecting configuration on constraint

Newton-Raphson algorithm

- ▶ $\mathbf{q}_0$ configuration,
- ▶ $f(\mathbf{q}) = 0$ non-linear constraint,
- ▶ ϵ numerical tolerance

Projection $(\mathbf{q}_0, f)$:

$\mathbf{q} = \mathbf{q}_0 ; \alpha = 0.95$

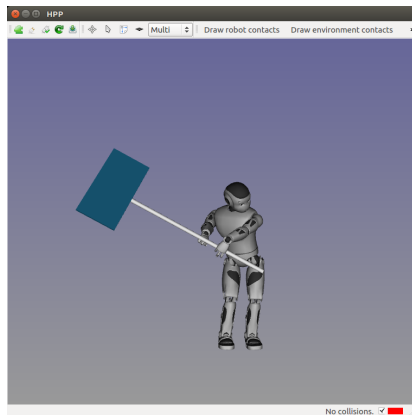
for i from 1 to max_iter :

$$\mathbf{q} = \mathbf{q} - \alpha \left(\frac{\partial f}{\partial \mathbf{q}}(\mathbf{q}) \right)^+ f(\mathbf{q})$$

if $\|f(\mathbf{q})\| < \epsilon$: return $\mathbf{q}$

return failure

Newton-Raphson algorithm

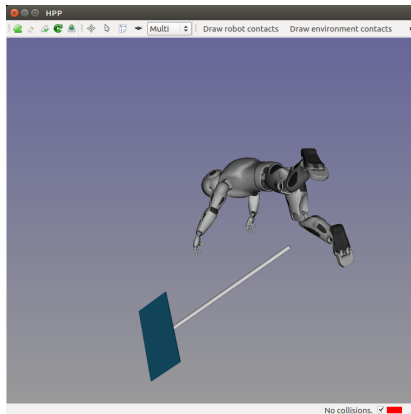


Goal : Generate a configuration satisfying the constraints.

Constraints

- ▶ quasi-static equilibrium (15)
- ▶ both hands hold the placard (10)

Newton-Raphson algorithm

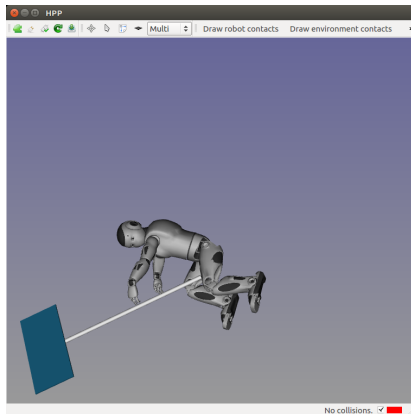


Shoot random configuration

Constraints

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- ▶ both hands hold the placard (10)

Newton-Raphson algorithm

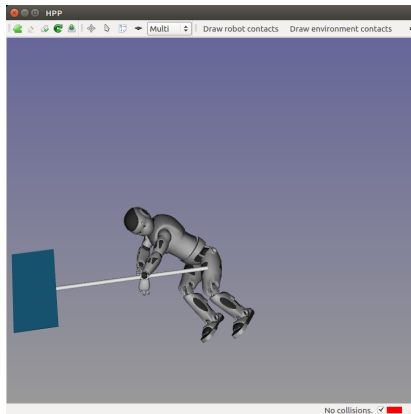


Solve linearized system

Constraints

- ▶ quasi-static equilibrium (15)
- ▶ both hands hold the placard (10)

Newton-Raphson algorithm

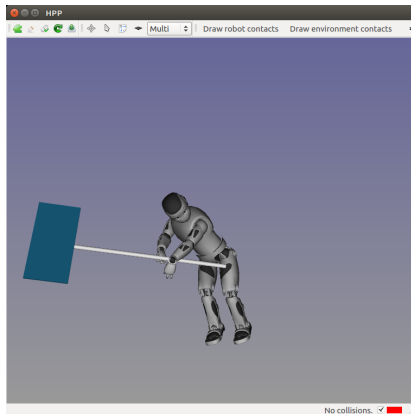


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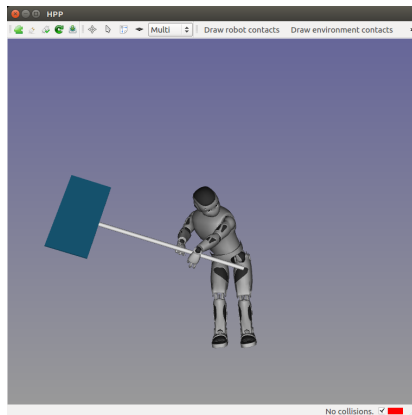


Solve linearized system

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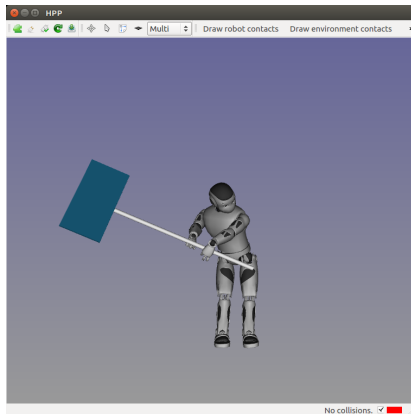


Solve linearized system

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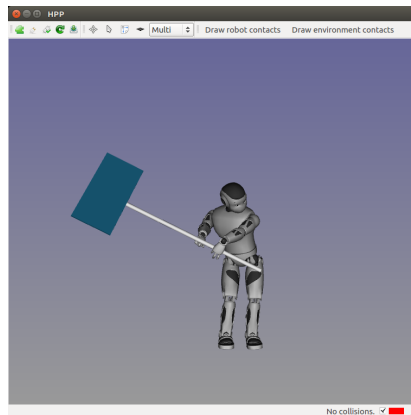


Solve linearized system

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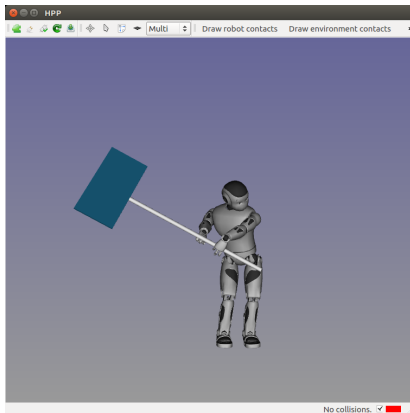


Solve linearized system

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- ▶ quasi-static equilibrium (15)
- ▶ both hands hold the placard (10)

Newton-Raphson algorithm



Constraints

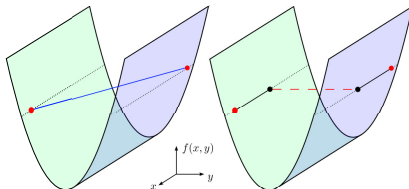
- ▶ quasi-static equilibrium (15)
- ▶ both hands hold the placard (10)

Result : a configuration that satisfies the constraints (up to given threshold).

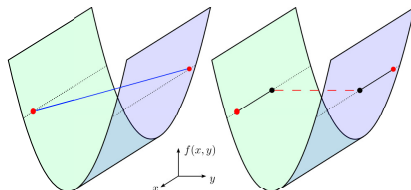
Projecting path on constraint

- ▶ $path$: mapping from $[0, 1]$ to $\mathcal{C}$
- ▶ $f(\mathbf{q}) = 0$ non-linear constraint,

Applying Newton Raphson at each point may result in a discontinuous path



Discontinuous Projection



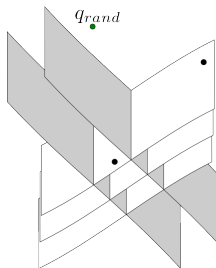
$$\mathcal{C} = \mathbb{R}^2, f(x, y) = y^2 - 1$$

$$\frac{\partial f}{\partial \mathbf{q}} = \begin{pmatrix} 0 & 2y \end{pmatrix}, \quad \frac{\partial f^+}{\partial \mathbf{q}} = \begin{pmatrix} 0 \\ \frac{1}{2y} \end{pmatrix} \text{ ou } \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$y_{i+1} = y_i + \frac{1 - y_i^2}{2y_i}$$

Algorithm

Manipulation RRT



Manipulation RRT

$\mathbf{q}_{rand} = \text{shoot_random_config}()$

for each connected component :

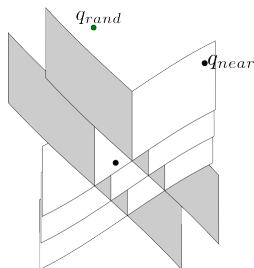
```
 $\mathbf{q}_{near} = \text{nearest\_neighbor}(\mathbf{q}_{rand}, \text{roadmap})$   
 $T = \text{select\_transition}(\mathbf{q}_{near})$   
 $\mathbf{q}_{proj} = \text{generate\_target\_config}(\mathbf{q}_{near}, \mathbf{q}_{rand}, T)$   
 $\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, T)$   
 $\text{roadmap.insert\_node}(\mathbf{q}_{new})$   
 $\text{roadmap.insert\_edge}(T, \mathbf{q}_{near}, \mathbf{q}_{new})$   
 $\text{new\_nodes.append}(\mathbf{q}_{new})$ 
```

for $\mathbf{q} \in (\mathbf{q}_{new}^1, \dots, \mathbf{q}_{new}^{n_{cc}})$:

```
connect ( $\mathbf{q}$ , roadmap)
```

Algorithm

Manipulation RRT



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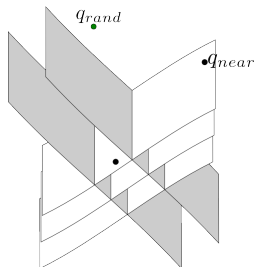
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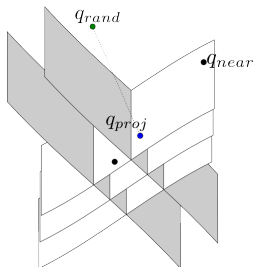
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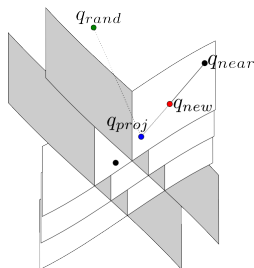
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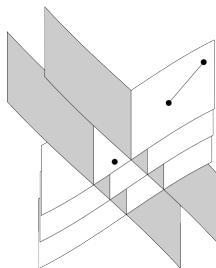
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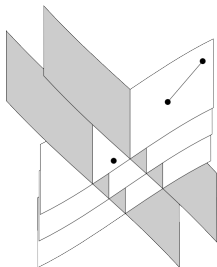
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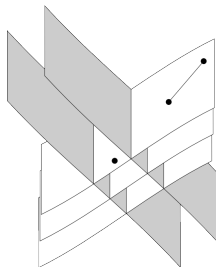
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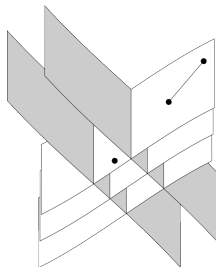
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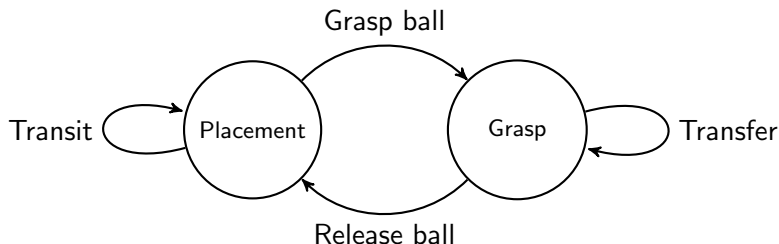
for $\mathbf{q} \in (\mathbf{q}_{new}^1, \dots, \mathbf{q}_{new}^{n_{cc}})$:

connect ($\mathbf{q}$, roadmap)

Select transition

$$T = \text{select_transition}(\mathbf{q}_{near})$$

Outward transitions of each state are given a probability distribution. The transition from a state to another state is chosen by random sampling.



Generate target configuration

$$\mathbf{q}_{proj} = \text{generate_target_config}(\mathbf{q}_{near}, \mathbf{q}_{rand}, T)$$

Once transition T has been selected, $\mathbf{q}_{rand}$ is *projected* onto the destination state S_{dest} in a configuration reachable by $\mathbf{q}_{near}$.

$$\begin{aligned} f_T(\mathbf{q}_{proj}) &= f_T(\mathbf{q}_{near}) \\ f_{S_{dest}}(\mathbf{q}_{proj}) &= 0 \end{aligned}$$

Extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, T)$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on transition constraint :

- ▶ if projection successful and projected path collision free

$$\mathbf{q}_{new} \leftarrow \mathbf{q}_{proj}$$

- ▶ otherwise $(\mathbf{q}_{near}, \mathbf{q}_{new}) \leftarrow$ largest path interval tested as collision-free with successful projection.

$$\forall \mathbf{q} \in (\mathbf{q}_{near}, \mathbf{q}_{new}), f_T(\mathbf{q}) = f_T(\mathbf{q}_{near})$$

Extend

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$$\forall \mathbf{q} \in (\mathbf{q}_{near}, \mathbf{q}_{new}), f_T(\mathbf{q}) = f_T(\mathbf{q}_{near})$$

Connect

connect ($\mathbf{q}$, roadmap)

for each connected component cc not containing $\mathbf{q}$:

for all n closest config $\mathbf{q}_1$ to $\mathbf{q}$ in cc :

▶ connect ($\mathbf{q}$, $\mathbf{q}_1$)

Connect

```
connect ( $\mathbf{q}_0, \mathbf{q}_1$ ) :  
     $S_0 = \text{state}(\mathbf{q}_0)$   
     $S_1 = \text{state}(\mathbf{q}_1)$   
     $T = \text{transition}(S_0, S_1)$   
    if  $T$  and  $f_T(\mathbf{q}_0) == f_T(\mathbf{q}_1)$  :  
        if  $p = \text{projected\_path}(T, \mathbf{q}_0, \mathbf{q}_1)$  collision-free :  
            roadmap.insert_edge( $T, \mathbf{q}_0, \mathbf{q}_1$ )  
    return
```

Relative positions as numerical constraints

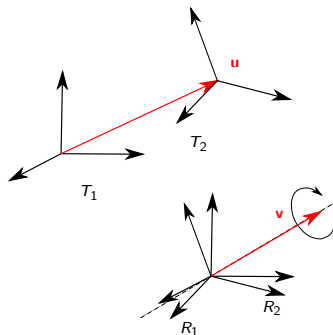
- ▶ $T_1 = T_{(R_1, t_1)} \in SE(3)$,
 $T_2 = T_{(R_2, t_2)} \in SE(3)$.
- ▶ $T_{2/1} = T_1^{-1} \circ T_2$ can be represented by a vector of dimension 6 :

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

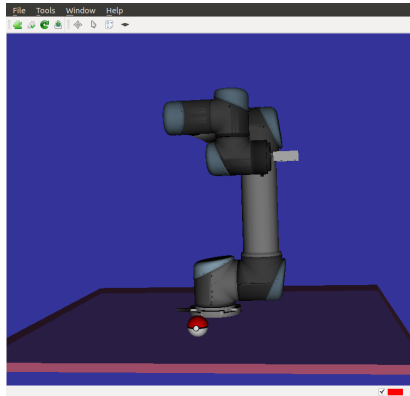
where

$$\mathbf{u} = R_1^T(t_2 - t_1)$$

$R_1^T R_2$ matrix of the rotation
around axis $\mathbf{v}/\|\mathbf{v}\|$ and of
angles $\|\mathbf{v}\|$.



A few words about the BE



► `script/grasp_ball.py`